Interference

In general, we've already seen this topic $\rightarrow$ Superposition!

$$\left.\begin{aligned} \psi_1(x,t) &= A\cos(k_0x - \omega_0 t) \\ \psi_2(x,t) &= A\cos(k_0x - \omega_0 t + \phi) \end{aligned}\right\} \text{monochromatic plane waves}$$

$$\psi_1 + \psi_2 = A\left[\cos(k_0x - \omega_0 t) + \cos(k_0x - \omega_0 t + \phi)\right]$$

$$= 2A\cos(\phi/2)\cos(k_0x - \omega_0 t + \phi/2)$$

[See Notes 4-3]

The form of interference here comes down to the phase difference, ϕ

<u>Constructive interference:</u> $\psi_1 + \psi_2 = 2A\cos(kx - \omega t)$; $\phi = 2\pi n$

<u>Destructive interference:</u> $\psi_1 + \psi_2 = 0$; $\phi = 2\pi n + 1$

The conditions for observing this in optical fields is a bit more strict

<u>Fresnel-Arago Laws:</u>

1) orthogonal, coherent polarization states <u>do not</u> interfere
2) parallel, coherent polarization states <u>do</u> interfere
3) Constituent, orthogonal polarization states of incoherent light does not interfere, even if rotated into alignment (ie: natural light)

For optical fields, $\omega \sim 10^{15}$ → Hard to monitor E-field!

Instead, monitor intensity/irradiance:

$$I_{Tot} = I_1 + I_2 + I_{12}$$

$$= \langle \vec{E}_1^2 \rangle_T + \langle \vec{E}_2^2 \rangle_T + 2 \langle \vec{E}_1 \cdot \vec{E}_2 \rangle_T$$

(Remember: interval T matters!)

$$\vec{E}_1 \cdot \vec{E}_2 = \underbrace{\vec{E}_{01} \cdot \vec{E}_{02}}_{\text{(this is where polarization matters)}} \underbrace{\cos(\vec{k}_1 \cdot \vec{r} - \omega t + \phi_1) \cos(\vec{k}_2 \cdot \vec{r} - \omega t + \phi_2)}_{\text{(this is where temporal/spatial coherence matters)}}$$

Coherence (what do we mean?)

- a formal definition requires mathematical tools we'll build up to later (autocorrelation & cross-correlation)
- for now, we mean that information about the field at one point <u>may</u> give us information about the field elsewhere

<u>temporal coherence:</u> (same point, different time)

$E(r,t)$ compared to $E(r, t+\Delta t)$

<u>Spatial coherence:</u>

$E(r,t)$ compared to $E(r+\Delta r, t)$

<u>Ex:</u> Laser Speckle
- high temporal coherence
- low spatial coherence

<u>Ex:</u> White light pt. source
- high spatial coherence
- low temporal coherence

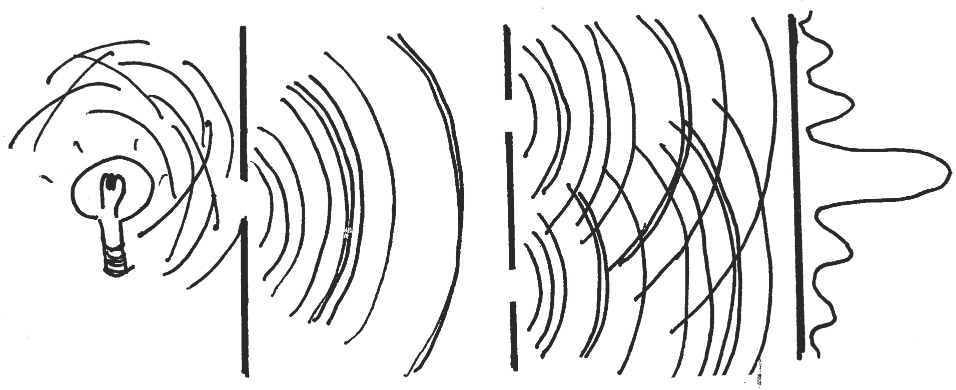
- <u>Spectral</u> coherence variously depends on temporal and spatial coherence (as was seen in HW#4)

Interferometers

- basically, a device for measuring the coherence and/or relative phase of two (or more) fields
- functionally, just send light from a source along two (or more) paths and observe the intensity of the combined fields
- are many, many ways of doing this – we'll look at a few common ones
- can effectively classify interferometers as either: ① Wavefront splitting ② Amplitude splitting

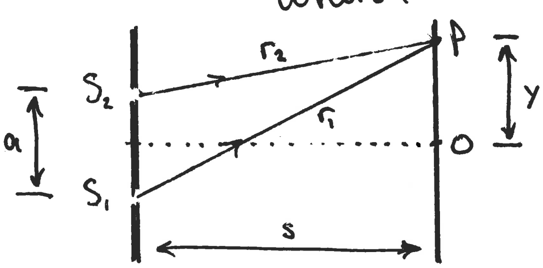
Wavefront-Splitting

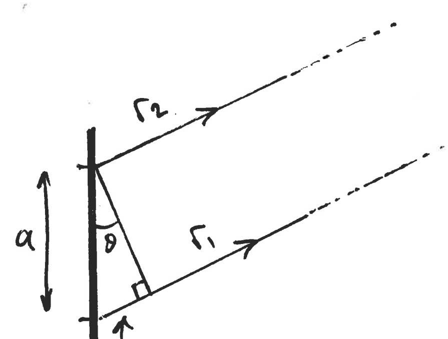
- requires spatial coherence
- Classic example: Young's Double Slit Experiment



measure intensity as a function of position

incoherent | spatially coherent | select wavefront at 2 positions





$$\Delta r = r_1 - r_2$$

$$\simeq a \sin\theta$$

$$\simeq a\theta \quad \text{(paraxial)}$$

$$r_1 = \sqrt{s^2 + \left(y + \frac{a}{2}\right)^2}$$

$$r_2 = \sqrt{s^2 + \left(y - \frac{a}{2}\right)^2}$$

$$\Rightarrow \Delta r \simeq a y / s$$

- Note: We'll talk a lot more about when it's okay to make these approximations when we get to diffraction

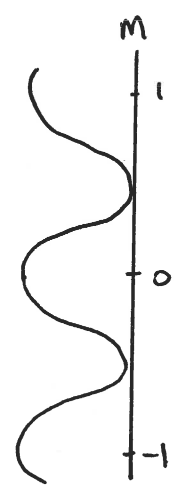
- there's now a phase difference between wavefronts due to the path-length difference: $\delta = k(r_1 - r_2)$

- as long as the intensities at each slit are the same, then

$$I = I_1 + I_2 + \sqrt{I_1 I_2} \cos\delta$$
$$= 2I_0 (1 + \cos\delta)$$
$$= 4I_0 \cos^2 \frac{\delta}{2} \quad \text{[half-angle identity]}$$

$$\Rightarrow I(y) = 4I_0 \cos^2\left(k(r_1 - r_2)\tfrac{1}{2}\right)$$
$$= 4I_0 \cos^2\left(\frac{ka}{2s} y\right)$$
$$= 4I_0 \cos^2\left(\frac{\pi a}{s\lambda} y\right)$$

[Note: fringes go away if the polarization is rotated, though]



- So there are bright "fringes" at: $y_m \simeq \frac{s}{a} m\lambda$

- this can be used to measure the wavelength of light!

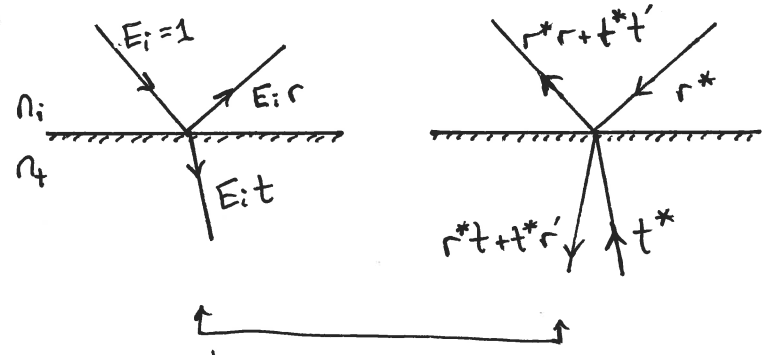
Amplitude-Splitting

- requires temporal coherence

- basic idea is to split some incident wave into separate beams, have them traverse different path lengths, and then recombine them

- natural at this point to make use of <u>Stokes' relations</u> [6-5]

(relations between reflection & transmission amplitude coeff's)



time-reversal invariance

$E e^{i(\omega t + \phi)} \rightarrow E e^{-i(\omega t+\phi)}$

- light from above: r, t
- light from below: r', t'

$\rightarrow \quad r^* r + t^* t' = 1$

$\quad\quad r^* t + t^* r' = 0$

- if these coeff's are real, then this collapses to

$\rightarrow \quad r^2 + tt' = 1$

$\quad\quad r' = -r$

[Note: have phase change when $n_i < n_t$, but not visa versa]

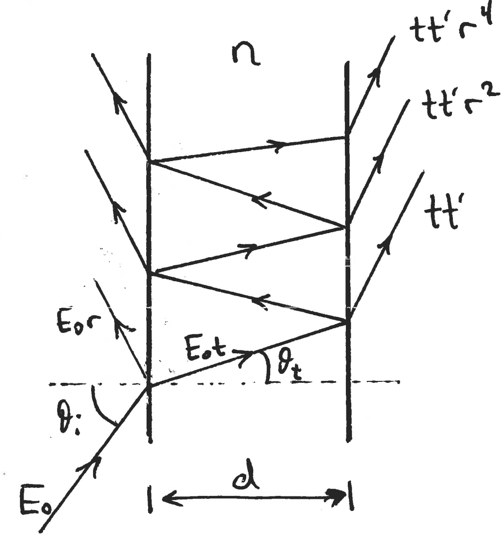
- a common example where this comes into play for interference is with the <u>Fabry-Perot</u> interferometer (or etalons)

- have light striking a flat glass plate that's partially silvered on both sides

@ first interface (air-glass): r, t

@ second interface (glass-air): r', t'

(if no absorption losses and symmetric, then $r' = -r$)



- the first transmitted beam is $E_0 tt'$

- the second transmitted beam traverses an additional distance that depends on d and $\theta_t = \sin^{-1}\left(\frac{1}{n}\sin\theta_i\right) \Longrightarrow$ accumulated phase shift $\delta = \frac{4\pi n d}{\lambda}\cos\theta_t$

- the total transmitted field is then a sum of componets

$$E_t = E_o tt' + E_o tt' r^2 e^{i\delta} + E_o tt' r^4 e^{i2\delta} + \dots$$

$$= E_o tt' \left(1 + r^2 e^{i\delta} + r^4 e^{2i\delta} + \dots\right)$$

$$= E_o \frac{tt'}{(1 - r^2 e^{i\delta})}$$

[a geometric series which converges]

- can then use Stokes' relations to simplify

$$E_t = E_o \frac{1-r^2}{(1-r^2 e^{i\delta})} = E_o \frac{1-R}{(1-Re^{i\delta})} = E_o \frac{T}{(1-Re^{i\delta})}$$

where $r^2 = R$ & $1 - R = T$

- we've calculated the transmitted field <u>amplitude</u>, but will probably be measuring irradiance/intensity

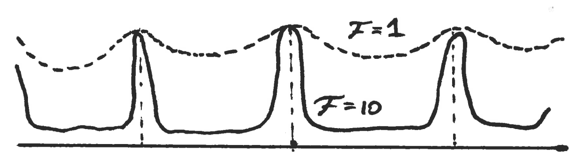
$$\Longrightarrow I_t \propto E_t E_t^* = I_i \frac{(1-r^2)^2}{(1+r^4) - 2r^2\cos\delta}$$

$\cos\delta = 1 - 2\sin^2(\delta/2)$

$$= \frac{I_i}{1 + \left(\frac{2r}{1-r^2}\right)^2 \sin^2(\delta/2)}$$

- then define "coefficient of finesse" : $\mathcal{F} \equiv \left(\frac{2r}{1-r^2}\right)^2$ [gets larger as $r \to 1$]

- now $\frac{I_t}{I_i} = \frac{1}{1 + \mathcal{F}\sin^2(\delta/2)}$, which is an Airy function



and see there are a series of resonance peaks that get sharper as $r \to 1$ ($\mathcal{F}$ gets larger)

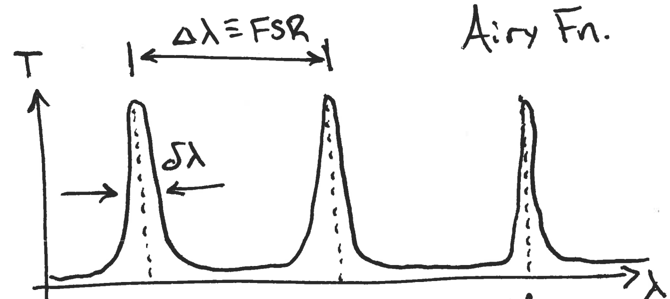
- Why do we do this? In this form the expression is a 2π periodic function of δ, parameterized by the coeff. of finesse, $\mathcal{F} = \left(\frac{2r}{1-r^2}\right)^2$

[Note: Only valid for a lossless cavity with identical mirrors]

- Using $I_R = I_o - I_t$, we also know the reflected power by the Fabry-Perot:

$$\frac{I_r}{I_o} = \frac{\mathcal{F}\sin^2(\delta/2)}{1+\mathcal{F}\sin^2(\delta/2)}$$

Airy Fn.



$\Delta\nu = \frac{c}{2nd}$, $\Delta\lambda = \frac{\lambda^2}{2nd}$

[index of refraction]

- We've seen the coeff of finesse pop up a few times now, but what is Finesse?

- first, need to know about Free Spectral Range (FSR), which is simply a characterization of round-trip time spent in an optical cavity (FSR normally given as $\Delta\nu$ or $\Delta\lambda$)

- also need some characterization of cavity quality factor, so use Full Width at Half Maximum (FWHM $\equiv \delta\lambda$)

- then Finesse is: $F = \frac{\Delta\lambda}{\delta\lambda}$, where $\Delta\lambda \simeq \frac{\lambda^2}{2nd\cos\theta_i}$ (not shown)

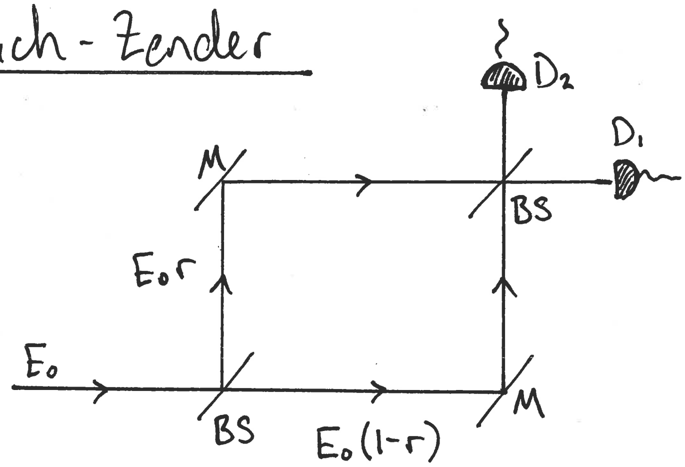
$$\Longrightarrow F = \frac{\Delta\lambda}{\lambda} \simeq \frac{\pi}{2\sin^{-1}(1/\sqrt{\mathcal{F}})} \simeq \frac{\pi\sqrt{\mathcal{F}}}{2} = \frac{\pi\sqrt{R}}{(1-R)} \quad (\text{for } R > \tfrac{1}{2})$$

where $\mathcal{F} = \left(\frac{2r}{1-r^2}\right)^2 = \frac{4R}{(1-R)^2}$

[See HW # 6-A2]

Other Amplitude - Splitting Interferometers

Mach-Zender



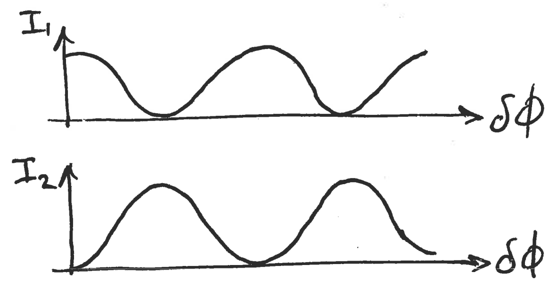
- if $r = \frac{1}{\sqrt{2}}$ & $R = \frac{1}{2}$

get in-phase component at D_1

$$I_1 = \frac{1}{2} I_0 (1 + \cos\delta\phi)$$

get out-of-phase component at D_2

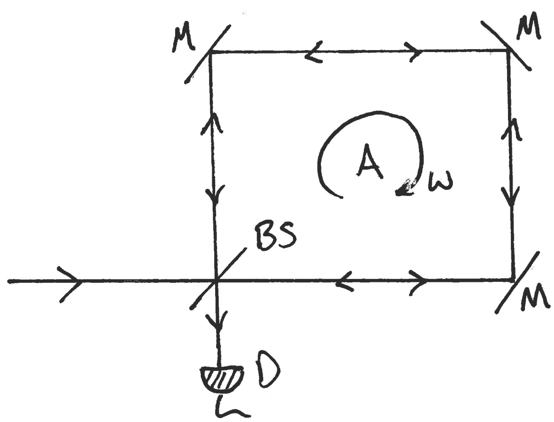
$$I_2 = \frac{1}{2} I_0 (1 - \cos\delta\phi)$$



- by placing a test object into one arm, can measure the phase shift due to path length difference (and therefore, index n)

[Sometimes very useful to record <u>both</u> parts]

Sagnac



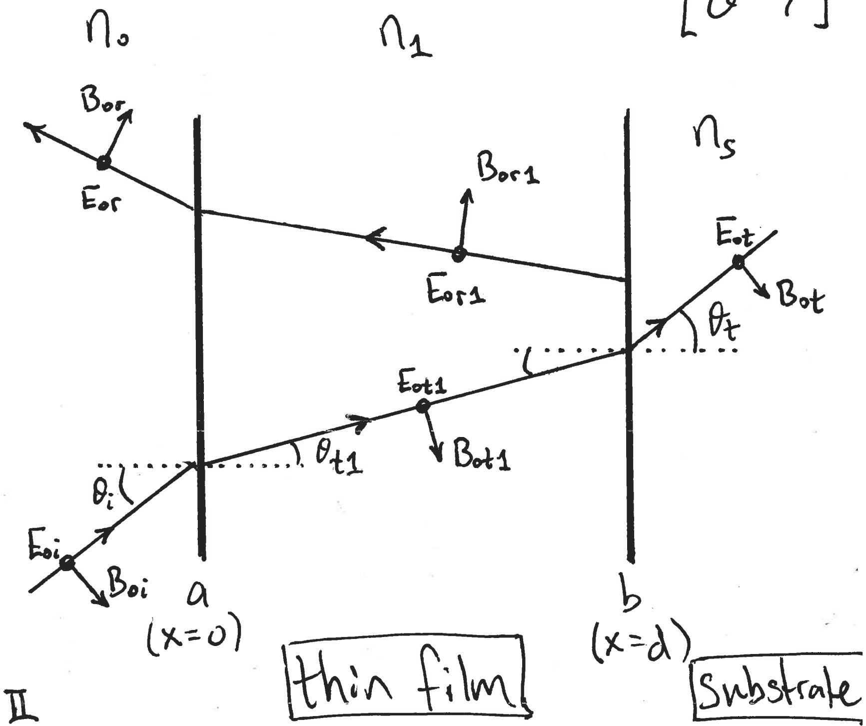
- if the interferometer is rotating, light has less distance to travel in one direction than the other

$$\Delta t = \frac{4A\omega}{c^2} \quad \& \quad \Delta\phi = \frac{2\pi c \Delta t}{\lambda}$$

- can be used to detect <u>very</u> small rotational speeds

Thin-Film Interference

- Start with a single thin film on some optical interface
- have multiple fields to deal with
  - 4 combine at interface I
  - 3 combine at interface II
  - will have to consider EM boundary conditions @ both I & II



<u>Incident Region</u>: $E_{oi}\, e^{i\vec{k}_i \cdot \vec{r}} + E_{or}\, e^{i\vec{k}_r \cdot \vec{r}}$

<u>Thin-Film Region</u>: $E_{ot1}\, e^{i\vec{k}_{t1} \cdot \vec{r}} + E_{or1}\, e^{i\vec{k}_{r1} \cdot \vec{r}}$

<u>Substrate Region</u>: $E_{ot}\, e^{i\vec{k}_t \cdot (\vec{r} - d\hat{x})}$ [shifted by $k \cdot d$]

<u>Note:</u> each of these fields represent the resultant of <u>all possible</u> waves (where the summation over all repeated E_r & E_t is already done)

- enforce boundary conditions: transverse E-field must be continuous across each interface (at a and b)

<u>interface a</u>: $E_a = E_{oi} + E_{or} = E_{ot1} + E_{or1}$

<u>interface b</u>: $E_b = E_{ot1}\, e^{i\delta} + E_{or1}\, e^{-i\delta} = E_{ot}$

where we account for the phase shift $\delta = (\vec{k}_{t1} \cdot \hat{x}) = \frac{2\pi}{\lambda_0} n_1 d \cos\theta_{t1}$ due to film thickness d.

We're matching phases everywhere, but are assuming small angles, so consider phases at constant y (straight across the thin film) [6-10]

- enforce boundary conditions: transvers B-field components also continuous

interface a: $B_a = B_{oi}\cos\theta_i - B_{or}\cos\theta_i = B_{ot1}\cos\theta_{t1} - B_{or1}\cos\theta_{t1}$

interface b: $B_b = B_{ot1}\cos\theta_{t1}\, e^{i\delta} - B_{or1}\cos\theta_{t1}\, e^{-i\delta} = B_{ot}\cos\theta_t$

Since $E/B = \frac{1}{\sqrt{\epsilon_o \mu_o}} = c$, can re-write these in terms of E-fields

interface a: $\alpha_i (E_{oi} - E_{or}) = \alpha_1 (E_{ot1} - E_{or1}) = B_a$

interface b: $\alpha_1 (E_{ot1} e^{i\delta} - E_{or1} e^{-i\delta}) = \alpha_s E_{ot} = B_b$

where $\alpha_i = \frac{n_i}{c}\cos\theta_i$, $\alpha_1 = \frac{n_1}{c}\cos\theta_{t1}$, $\alpha_s = \frac{n_s}{c}\cos\theta_t$

- now can relate E_a and E_b: add & subtract BC's for these

$$\Longrightarrow E_{ot1} = \frac{\alpha_1 E_b + B_b}{2\alpha_1} e^{-i\delta} \quad \& \quad E_{or1} = \frac{\alpha_1 E_b - B_b}{2\alpha_1} e^{i\delta}$$

So using these definitions of E_a and B_a in terms of the in-film fields

$$E_a = E_{ot1} + E_{or1} = \cos\delta\, E_b - \frac{iB_b}{\alpha_1}\sin\delta$$

$$B_a = \alpha_1 (E_{ot1} - E_{or1}) = -i\alpha_1 E_b \sin\delta + B_b\cos\delta$$

- Can rewrite this in matrix form

thin film transfer Matrix T

$$\begin{bmatrix} E_a \\ B_a \end{bmatrix} = \begin{bmatrix} \cos\delta & -\frac{i}{\alpha_1}\sin\delta \\ -i\alpha_1 \sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} E_b \\ B_b \end{bmatrix}$$

$\delta = \frac{2\pi}{\lambda_o} n d \cos\theta_t$

$\alpha_1 = \frac{n_1}{c}\cos\theta_t$

$\theta_t = \sin^{-1}\left(\frac{n_i}{n_s}\sin\theta_i\right)$

- multiple thin-film layers can now be represented as ordered products:

$$\begin{bmatrix} E_a \\ B_a \end{bmatrix} = T_1 T_2 T_3 \cdots T_N \begin{bmatrix} E_N \\ B_N \end{bmatrix}$$

Single-Layer Antireflection Coating

- the transfer matrix has the form: $T = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos\delta & -\frac{1}{\alpha_1}\sin\delta \\ -i\alpha_1 \sin\delta & \cos\delta \end{bmatrix}$

- can then write the reflection amplitude coefficient due to film as

$$r_{film} = \frac{\alpha_i A + \alpha_i \alpha_s B - C - \alpha_s D}{\alpha_1 A + \alpha_i \alpha_s B + C + \alpha_s D}$$

for reference: $t_{film} = \dfrac{2\alpha_i}{\alpha_i A + \alpha_i \alpha_s B + C + \alpha_s D}$

$$r_{film} = \frac{\alpha_i \cos\delta - i\frac{\alpha_i \alpha_s}{\alpha_1}\sin\delta + i\alpha_1 \sin\delta - \alpha_s \cos\delta}{\alpha_i \cos\delta - i\frac{\alpha_i \alpha_s}{\alpha_1}\sin\delta - i\alpha_1 \sin\delta + \alpha_s \cos\delta}$$

$$= \frac{n_1(n_i - n_s)\cos\delta - i(n_i n_s - n_1^2)\sin\delta}{n_1(n_i + n_s)\cos\delta - i(n_i n_s + n_1^2)\sin\delta}$$

normal incidence

$\alpha_i = \dfrac{n_i}{c}$

$\alpha_1 = \dfrac{n_1}{c}$

$\alpha_s = \dfrac{n_s}{c}$

$$R = |r|^2 = \frac{n_1^2(n_i - n_s)^2\cos^2\delta + (n_i n_s - n_1^2)^2 \sin^2\delta}{n_1^2(n_i + n_s)^2\cos^2\delta + (n_i n_s + n_1^2)^2 \sin^2\delta}$$

- for a film with thickness $d = \frac{\lambda}{4} = \frac{\lambda_0}{4n_1}$, the phase shift is

$\delta = \frac{2\pi}{\lambda_0} n_1 d \cos\theta_{t1} = \frac{\pi}{2}$ (180°! - destructive interference) with surface reflection

- So because $\cos\delta = 0$ and $\sin\delta = 1$, then

$$R = \left(\frac{n_i n_s - n_1^2}{n_i n_s + n_1^2}\right)$$

- Compare to Fresnel Eqn for only 1 surface (at normal incidence)

$$R_o = \frac{n_i - n_s}{n_i + n_s}$$

- Note that the reflectance completely vanishes if the film index is

$$R \to 0, \text{ when } n_1 = \sqrt{n_i n_s}$$

- for an air-glass interface ($n_i = 1$, $n_s = 1.5$), the ideal film index for this effect is $n_1 \sim 1.2$. There are no materials with this index that are useful for optics → instead, MgF_2 is usually used ($n = 1.38$)

"bare" air-glass interface: $R = \frac{1 - 1.5}{1 + 1.5} \simeq 4\%$

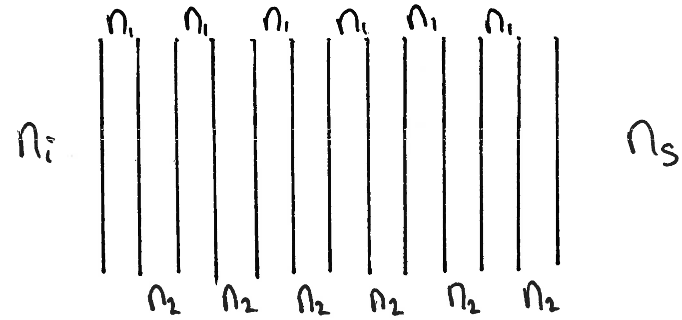
MgF_2 AR coating: $R = \frac{(1 \cdot 1.5 - 1.38^2)}{(1 \cdot 1.5 + 1.38^2)} \simeq 1\%$ (better... not perfect)

- in actuality, R will vary depending on λ, θ_i, d
(an AR coating optimized for $\lambda = 633$ nm @ 0° incidence is <u>not</u> going to work as well at 532 nm or at 45° incidence)

Multi-Layer High-Reflector Dielectric Mirrors

- Using only transparent materials, it's actually possible to make a mirror much more reflective than from metal

- alternate 2 dielectric thin films



- if the ratio $\frac{n_2}{n_1}$ is very different from $\sqrt{n_2/n_1}$, then get fairly strong reflection ($T \sim 0$ into substrate, n_s)

- for N of these double layers (alternating high-low index)

$$R = \left[\frac{(n_i/n_s)\left(\frac{-n_2}{n_1}\right)^{2N} - 1}{(n_i/n_s)(-n_2/n_1)^{2N} + 1} \right]^2$$

Ex: Say, air-glass, so $\frac{n_i}{n_s} = \frac{1}{1.5}$ and that $\frac{n_2}{n_1} = \frac{1}{1.27}$

$\Longrightarrow$ $N = 2$: $R = 36\%$

$N = 4$: $R = 68\%$

$N = 12$: $R = 99\%$