Physics 47 — Optics

Exam 1 – Friday, October 6, 2017

Allowed items: One 8.5” x 11” formula sheet; textbook; lecture notes; calculator.

Show all work for full credit.

Use a separate sheet of paper for solution to each problem.

Write your name on every sheet of solutions.

When handing in, verify the order of solutions.

Solve any three problems.

Problem 1: Monochromatic EM Wave Propagation

Consider a monochromatic electromagnetic plane wave passing through a dielectric medium, where the electric field component of the wave can be described (in MKS units) by

$$\mathbf{E}(z,t) = 3\cos\left[(6\pi\times10^{6})\ z + (12\pi\times10^{14})\ t\right]\hat{\mathbf{x}}\ +\ 4\cos\left[(6\pi\times10^{6})\ z + (12\pi\times10^{14})\ t + \pi\right]\hat{\mathbf{y}}$$

(a) In what direction is the wave propagating?

(b) What is the angular frequency (rad/sec) and “linear” frequency (Hz) of the wave?

(c) What is the phase velocity of the wave in the dielectric?

(d) What is the wavelength in the dielectric medium and what would it be in vacuum?

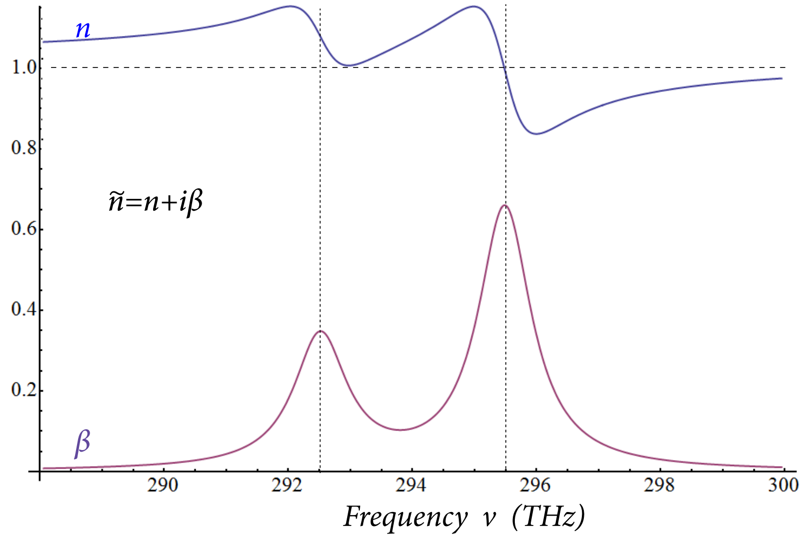
(e) What is the index of refraction of the dielectric medium?

(f) What is the magnitude of the electric field at $t = 2 \times 10^{-14}$ s and $z = 5 \times 10^{-6}$ m?

Problem 2: Near-Resonant Light-Matter Interactions

The real and imaginary parts of the index of refraction for a hypothetical dielectric material are show below. The material has resonances at two frequencies, $\nu_{0a} = 293$ THz and $\nu_{0b} = 295$ THz.

Answer the following questions by inspecting the figure and giving estimated values as appropriate (e.g. give frequencies rounded to the nearest integer value).



(a) At what frequency(ies) is the phase velocity (v_p) the smallest?

(b) At what frequency(ies) is v_p the largest and how does it compare with the speed of light?

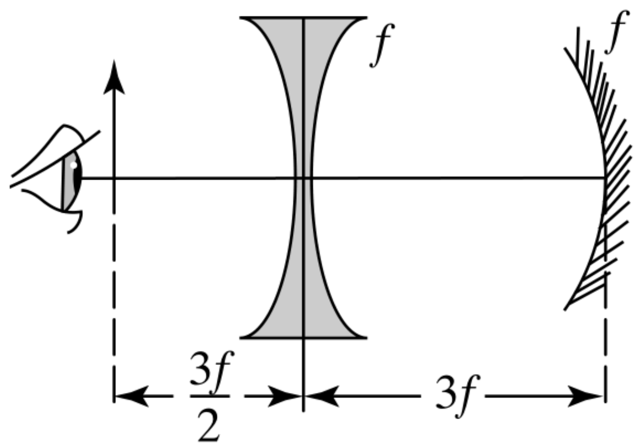
(c) Over what frequency range on the graph could you observe total internal reflection at an interface going from the medium into air?

(d) At what frequency does the strongest absorption occur?

(e) To what depth will a wave of frequency from part (d) penetrate into this material before its irradiance is reduced by a factor of $1/e$?

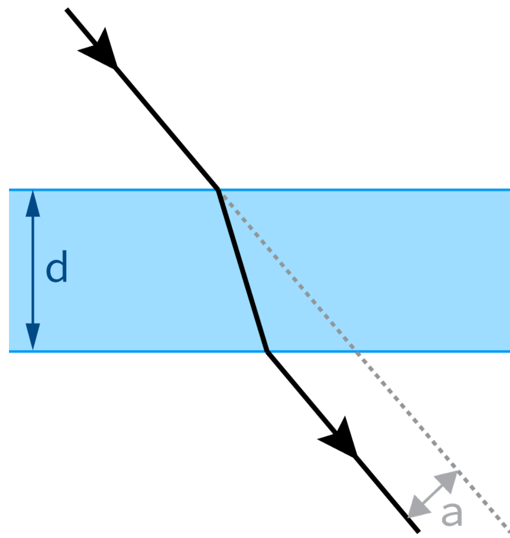
Problem 3: Geometric Optics

A diverging thin lens and a concave mirror have focal lengths of equal *magnitude*. An object is placed $(3/2)f$ from the diverging lens, and the mirror is placed a distance $3f$ on the other side of the lens. Rays from the object therefore undergo subsequent refraction, reflection, and refraction in this lens-mirror system. Using Gaussian optics, calculate the following:



- For rays initially coming from the object:
  - (a) What is the image distance, s_i, from the first lens traversal?
  - (b) Is this image real or virtual?
  - (c) What is the transverse magnification, M_T, for this first traversal of the lens?
- After passing the first lens, the light rays impinge on the mirror.
  - (d) How far away from the mirror does the object appear? (i.e.: what is s_o for the mirror?)
  - (e) For the image caused by the mirror, what is the image distance, s_i?
  - (f) What is the transverse magnification, M_T, due to just the mirror?
- After reflection from the mirror, the light rays make a second pass through the lens from the other side.
  - (g) How far away from the lens does the object now appear?
  - (h) What is the image distance, s_i, from this second lens traversal?
  - (i) What is the transverse magnification, M_T, due to this second lens traversal?
  - (j) What is the total magnification, M_T^{tot}, of the lens-mirror system?

Problem 4: EM Waves Incident on Dielectric Media



(a) Show that a ray incident at θ_i to a planar glass pane immersed in air will emerge from the glass' backside at the same angle.

(b) Determine the values of the amplitude reflection coefficients for light incident at 30° on the first air-glass interface ($n_{glass} = 1.5$). Assume a beam of unpolarized (i.e. randomly polarized) light.

(c) What are the phase changes of each polarization component of the reflected beam with respect to the incident beam?

(d) What are the transmittances into the glass of each polarization component of the beam? Additionally, what is the total transmittance into the glass?

(e) Calculate the critical angle beyond which there is total internal reflection at the internal glass-air interface.

(f) Derive an expression for the displacement a of the ray if the thickness of glass is d.

(g) Show that the expression you found in (f) becomes $a \approx d(\theta_i - \theta_t)$ under the paraxial approximation. [Not essentially needed, but perhaps helpful, is the identity: $sin(\alpha - \beta) = sin\,\alpha\; cos\,\beta - sin\,\beta\; cos\,\alpha$]