Exam 1 Solutions

1.

$$E(z,t) = 3\cos\left[(6\pi E6)z + (12\pi E14)t\right]\hat{x} + 4\cos\left[(6\pi E6)z + (12\pi E14)t + \frac{\pi}{2}\right]\hat{y}$$

a) $-\hat{z}$ - direction

b) $k = 6\pi E6 \longrightarrow \lambda = \frac{2\pi}{k} = \frac{1}{3}E\text{-}6$ (meters)

$$\boxed{\omega = 12\pi E14 \text{ (radians/sec)} \longrightarrow \nu = \frac{\omega}{2\pi} = 6E14 \text{ (Hz)}}$$

c) $$V_p = \frac{-\left(\frac{\partial E}{\partial t}\right)}{\left(\frac{\partial E}{\partial z}\right)} = \frac{-\omega}{k} = \frac{-\left(12\pi E14 \frac{rad}{sec}\right)}{\left(6\pi E6 \frac{rad}{m}\right)} = 2E8 \frac{m}{s}$$

d) in medium: $V_p = \frac{\omega}{k} = \nu\lambda$ $\quad\left(\lambda = \frac{1}{3}E\text{-}6 \text{ meters}\right)$

in vacuum: $c = \frac{\omega}{k_0} = \nu\lambda_0$

ν remains const. $\Longrightarrow \nu = \frac{c}{\lambda_0} = \frac{V_p}{\lambda}$

$$\Longrightarrow \lambda_0 = \frac{c\lambda}{V_p} = \frac{(3E8 \frac{m}{s})(\frac{1}{3}E\text{-}6\, m)}{(2E8\, m/s)} = \frac{1}{2}E\text{-}6\, m$$

e) $$n = \frac{c}{v} = \frac{(3E8\, m/s)}{(2E8\, m/s)} = 3/2$$

f) $$E = (3\hat{x} + 4e^{i\frac{\pi}{2}}\hat{y})\, e^{i\left[(6\pi E6 \frac{1}{m})(5E\text{-}6\, m) + (12\pi E14 \frac{1}{s})(2E\text{-}14\, s)\right]}$$

$$= (3\hat{x} + 4i\hat{y})\, e^{i\left[(30\pi) + (24\pi)\right]} = (3\hat{x} - 4\hat{y})\, e^{i54\pi}$$

Switch back to $Re\{E\} = 3\cos(54\pi)\,\hat{x}$

$$= 3\hat{x}$$

2/

a) $V_p = \frac{c}{n}$, so V_p is min when n is max @ ~ 292 THz AND 295 THz

b) similarly, V_p is max @ ~ 296 THz where $n \simeq 0.8 \Rightarrow V_p = \frac{c}{0.8} = 1.25c$ (!)

c) necessary condition for TIR is $n > n_{air} \simeq 1$.

So, basically all freq's below 295.5 THz

d) have max absorption when imaginary part of index is largest

$\Rightarrow \beta \simeq 0.65$ @ 295.5 THz

e) [see Hecht 4.8 - pg 131]

Can write the wave as $E(x,t) = E_0 e^{i(kx - \omega t)} = E_0 e^{i\omega(\frac{k}{\omega} - t)}$

and remember that $\tilde{n} = \frac{c}{v}$, where $v = \frac{\omega}{k} = \frac{\tilde{n}}{c}$

Now, $E(x,t) = E_0 e^{i\omega(\frac{\tilde{n}x}{c} - t)} = E_0 e^{i\omega[\frac{x}{c}(n + i\beta) - t]}$

$$= E_0 e^{i\omega \frac{x}{c} n} e^{i\omega \frac{x}{c} i\beta} e^{-i\omega t} = E_0 e^{-\frac{\omega}{c}\beta x} e^{i\omega(\frac{x}{c}n - t)}$$

then, since $I \propto E^2$, have $E^2(x,t) = E_0^2 e^{-\frac{2\omega}{c}\beta x} e^{2i\omega(\frac{x}{c}n - t)}$

So I will decrease by $\frac{1}{e}$ at $x = \frac{c}{2\omega\beta} = \frac{(\frac{1}{2\pi})(2.998 E8\ m/s)}{2(295.5\ THz)(0.65)} = 124\ nm$

3/

a) $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{-f_L} \Rightarrow \frac{1}{s_i} = \frac{1}{-f_L} - \frac{1}{s_o} = \frac{1}{-f} - \frac{1}{\frac{3}{2}f} = -\left(\frac{1}{f} + \frac{2}{3f}\right) = -\frac{5}{3f}$

$\Rightarrow \underline{\underline{s_i = -\frac{3f}{5}}}$

b) image appears left of lens $\rightarrow$ <u>virtual</u>

c) $M_T^a = \frac{-s_i}{s_o} = -\frac{(-3/5\ f)}{(3/2\ f)} = \underline{\underline{\frac{2}{5}}}$ (erect)

d) $s_o' = -s_i + 3f = \frac{3}{5}f + 3f = \underline{\underline{\frac{18}{5}f}}$

e) $\frac{1}{s_o'} + \frac{1}{s_i'} = \frac{1}{+f} \Rightarrow \frac{1}{s_i'} = \frac{1}{f} - \frac{1}{s_o'} = \frac{1}{f} - \frac{5}{18f} = \frac{13}{18f} \Rightarrow \underline{\underline{s_i' = \frac{18f}{13}}}$

f) $M_T^b = \frac{-s_i'}{s_o'} = \frac{-\left(\frac{18}{13}f\right)}{\left(\frac{18}{5}f\right)} = \underline{\underline{-\frac{5}{13}}}$ (inverted)

g) $s_o'' = 3f - s_i' = 3f - \frac{18}{13}f = \underline{\underline{\frac{21}{13}f}}$

h) $\frac{1}{s_i''} = \frac{1}{-f} - \frac{1}{s_o''} = -\left(\frac{1}{f} + \frac{13}{21f}\right) = -\frac{34}{21f} \Rightarrow \underline{\underline{s_i'' = \frac{-21f}{34}}}$

i) $M_T^c = \frac{-s_i''}{s_o''} = -\frac{\left(\frac{-21}{34}f\right)}{\left(\frac{21}{13}f\right)} = \underline{\underline{\frac{13}{34}}}$ (erect)

j) $M_T^{tot} = M_T^a M_T^b M_T^c = \left(\frac{2}{5}\right)\left(\frac{-5}{13}\right)\left(\frac{13}{34}\right) = \underline{\underline{\frac{-1}{17}}}$ (inverted)

4/

a) $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$, where $n_1 = n_3 = 1$, $n_2 = n_{glass} = \frac{3}{2}$

$$\Longrightarrow \sin\theta_1 = \sin\theta_3 \Longrightarrow \underline{\theta_1 = \theta_3}$$

b) $$r_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_t \cos\theta_i + n_i \cos\theta_t} \quad \& \quad r_{\perp} = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t}$$

where $\theta_t = \sin^{-1}\left(\frac{\sin\theta_i}{n_t}\right) = \sin^{-1}\left(\frac{\sin 30^\circ}{1.5}\right) = 19.5^\circ$

then $$r_{\parallel} = \frac{1.5\cos(30^\circ) - \cos(19.5^\circ)}{1.5\cos(30^\circ) + \cos(19.5^\circ)} = \underline{0.159}$$

$$r_{\perp} = \frac{\cos(30^\circ) - 1.5\cos(19.5^\circ)}{\cos(30^\circ) + 1.5\cos(19.5^\circ)} = \underline{-0.240}$$

c) Remember: $r_{\parallel} = \left(\frac{E_{or}}{E_{oi}}\right)_{\parallel}$ & $r_{\perp} = \left(\frac{E_{or}}{E_{oi}}\right)_{\perp}$

so can see for p-polarization: 0° phase change
s-polarization: 180° phase change

d) $$R_{\parallel} + T_{\parallel} = 1 \Longrightarrow T_{\parallel} = 1 - (0.159)^2 = \underline{0.975}$$

$$R_{\perp} + T_{\perp} = 1 \Longrightarrow T_{\perp} = 1 - (-0.240)^2 = \underline{0.9424}$$

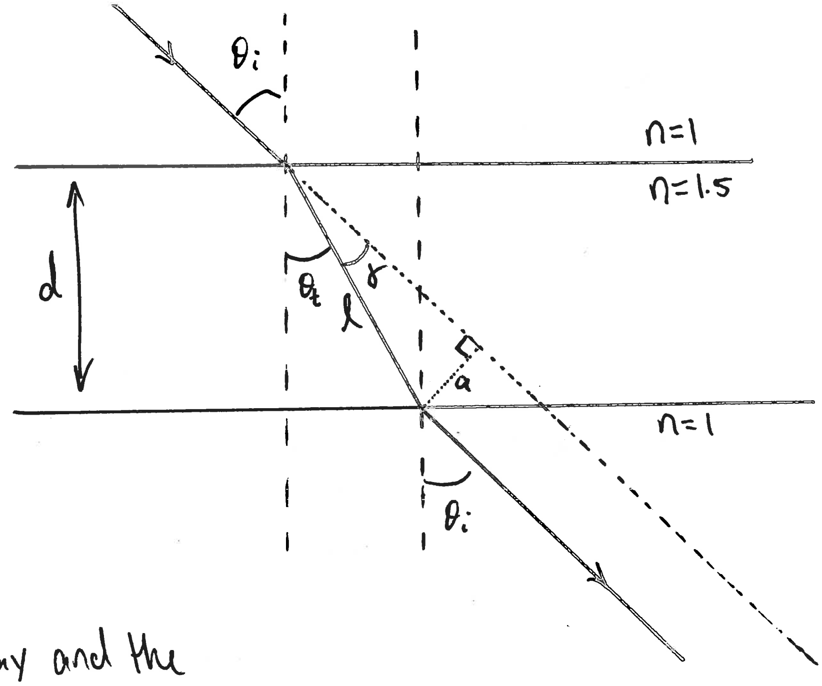
$$T = \frac{1}{2}(T_{\parallel} + T_{\perp}) = \underline{0.9587}$$

e) condition for critical angle: $n_{glass} \sin\theta_c = \sin 90^\circ = 1$

$$\Longrightarrow \theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = \underline{41.8^\circ}$$

4 cont.

f) the thickness of the glass pane can be written in terms of the path length within the glass as:



$$d = l\cos\theta_t$$

the angle between the initial ray and the refracted ray is:

$$\gamma = \theta_i - \theta_t$$

can then get the displacement as:

$$a = l\sin\gamma = \underline{\underline{\left(\frac{d}{\cos\theta_t}\right)\sin(\theta_i - \theta_t)}}$$

$$= \left(\frac{d}{\cos\theta_t}\right)\left(\sin\theta_i\cos\theta_t - \cos\theta_i\sin\theta_t\right)$$

Sum-difference trig identity

$$= d\left(\sin\theta_i - \cos\theta_i\tan\theta_t\right)$$

then in paraxial approximation: $(\sin x \simeq \tan x \simeq x$ & $\cos x \simeq 1)$

$$\Longrightarrow \underline{\underline{a \simeq d(\theta_i - \theta_t)}}$$