Physics 47 — Fall 2017

Problem Set 1
Due Wed. Sept 20, 2017
(before start of class)

Textbook Problems

1. Hecht 2.18 [4 pts.]
   This is a simple exercise to make sure you can connect the graphical representation of a wave to its mathematical representation.

2. Hecht 2.36 [4 pts.]
   This is to illustrate how the general notion of a wave velocity applies to something more pulse-like than wave-like. Note: Gaussian pulses will reappear frequently in this course, we might as well start off with one.

3. Hecht 2.44 [6 pts.]
   A little practice to help make sure you're comfortable with the notion of magnitude of a complex-valued quantity.

4. Hecht 3.4 [4 pts.]
   Reinforcing the origin of the very important constraint that $\vec{E} = c\vec{B}$ in an electromagnetic wave.

5. Hecht 3.16 [6 pts.]
   This is your first "real" homework problem, the rest have been really easy so far. This one should help you brush up your calculus skills after the summer. It may also be your first direct encounter with the "sinc" function. Advisory: trig identities are needed to coerce this expression into shape.

6. Hecht 3.24 [4 pts.]
   Time for calculation with some actual numbers to get an idea of the magnitude of quantities in a given situation. Be sure to give your answer in SI units! (V/m or N/C, okay.)

7. Hecht 3.42 [4 pts.]
   You may have some innate sense of the energy carried by light, but not its momentum. You can easily guess the force it exerts is small, the question is, how small? You also need to correctly account for absorption versus reflection. (Advisory: this problem gets even more interesting if the light isn't at normal incidence. I won't ask you to calculate that.. yet.)

Additional Problems

A1. [4 pts.] – Using the complex exponential representation of the trigonometric functions

$$\cos\phi = \frac{e^{i\phi} + e^{-i\phi}}{2}, \ \sin\phi = \frac{e^{i\phi} - e^{-i\phi}}{2i},$$

prove the identity:

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta.$$

This can be done in about 4 lines.

A2. [6 pts.] – Prove that the spherical harmonic wave function

$$\psi(r,t) = \frac{1}{r}e^{i(kr-\omega t)}$$

is a solution of the Helmholtz (wave) equation in spherical coordinates. (See Section 2.9 of Hecht for the form of the Laplacian operator in spherical coordinates.)

(This one is a bit plug-and-chug, but will warm up any cool memory of how to take derivatives.)

A3. [8 pts.] – Show that the average $\langle fg \rangle$ over exactly one period $T = 2\pi/\omega$ of the product of two harmonic functions $f(\mathbf{r},t) = A\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta_a)$ and $g(\mathbf{r},t) = B\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \delta_b)$ is given in complex notation by

$$\langle fg \rangle = \tfrac{1}{2}\mathrm{Re}(\tilde{f}\tilde{g}^*),$$

where $\tilde{f} = Ae^{i\delta_a}$ and $\tilde{g} = Be^{i\delta_b}$.

Hint: You'll need to split the expression into time-dependent and time-independent parts, and correctly figure out which terms vanish after integration over one period. You can choose to do this problem using trig functions or complex exponentials; I personally find the latter to be cleaner, but full marks for it either way. Some of you might choose to see if you can do it both ways (no extra credit though, sorry).