Physics 47 — Fall 2017

Problem Set 2
Due Wed. Sept 27, 2017
(before start of class)

Textbook Problems

1. Hecht 4.4 [5 pts.]
   Here, you'll go through the math I outlined in class to show the form of the response of a driven damped harmonic oscillator, and more particularly, the expression for the phase lag as a function of the drive frequency.

2. Hecht 4.7 [2 pts.]
   This is a somewhat simple warm-up geometry problem to get you thinking about how to approach problems that involve reflection from planar surfaces.

3. Hecht 4.28 [4 pts.]
   Now we switch to the geometry of refraction. From experience, you already know that the apparent depth of an object is affected by the index of the medium it's in. Now you'll quantify this. A trick is needed here, though: **assume that the angle of incidence on the air-glass interface is small** ($\sin\theta \approx \theta$) This is called the "paraxial approximation" and we'll use it more in chapters 5 and 6.

4. Hecht 4.51 [4 pts.]
   The Fresnel equations can take a variety of different forms, you will need to know how to recast them into the most useful form for a given problem.

5. Hecht 4.81 [3 pts.]
   This is a fairly simple problem about the critical angle in a fairly extreme case: diamond.

Additional Problems

A1. [4 pts.] – The air-glass interface is the most common interface configuration encountered in optics. Use a computer (Matlab, Mathematica, Python, etc.) to plot the following sets of reflection and transmission coefficients as a function of angle, assuming $n_i = 1$ for air and $n_t = 1.6$ for glass. Explicitly label Brewster's angle in all appropriate plots. The sheets you hand in should show the functions plotted (written out or in code form is fine).

(a) $r_\parallel$ and $t_\parallel$ (plot together on the same graph)
(b) $R_\parallel$ and $T_\parallel$ (plot together on the same graph)
(c) $r_\perp$ and $t_\perp$ (plot together on the same graph)
(d) $R_\perp$ and $T_\perp$ (plot together on the same graph)

A2. [4 pts.] – One usually thinks of Brewster's angle for a transparent material, but a similar phenomenon happens in metals. Find Brewster's angle for a silver mirror by calculating the reflectance for $R_{\parallel}$ and finding its minimum. The *complex* index of refraction of silver for visible light is $\tilde{n} = n + i\kappa = 0.13 + 4.0i$. Again, you will want to use a computer program to do this calculation (Matlab, Mathematica, Python, etc.). You may use a simple "plot the function, then plug in numbers close to what looks like a minimum" approach. If you're feeling a little more ambitious, try using a numerical minimization technique. Either way, generate a plot of the function and document what you do clearly, whatever it is.

A3. [6 pts.] – In a gas, the relative dielectric constant $\epsilon_r(\omega) = \epsilon(\omega)/\epsilon_0$ is very nearly equal to one. It can then be shown that the index of refraction $n(\omega)$ and absorption coefficient $\alpha(\omega)$ are related to the real and imaginary parts of ϵ_r as follows:

$$n(\omega) = \sqrt{\mathrm{Re}\{\epsilon_r(\omega)\}} \approx 1 + \frac{Ne^2}{2m\epsilon_0} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$$

$$\alpha(\omega) \approx k\,\mathrm{Im}\{\epsilon_r(\omega)\} \approx \frac{kNe^2}{m\epsilon_0} \frac{\gamma\omega}{(\omega_0^2 - \omega^2)^2 + \gamma^2\omega^2}$$

When the frequency of the electromagnetic wave is close to the resonance frequency:

$$\omega \approx \omega_0, \text{ and } \Delta\omega = \omega_0 - \omega \ll \omega, \omega_0$$

we can simplify the formulas for $n(\omega)$ and $\alpha(\omega)$ using the following two approximations:

$$\omega_0^2 - \omega^2 = (\omega_0 + \omega)(\omega_0 - \omega) \approx 2\omega_0(\omega_0 - \omega) = 2\omega_0\Delta\omega$$

$$\gamma\omega \approx \gamma\omega_0$$

(a) Show that under these approximations $n(\omega)$ and $\alpha(\omega)$ take the following form:

$$n(\omega) \approx 1 + \frac{Ne^2}{4m\epsilon_0\omega_0} \frac{\Delta\omega}{(\Delta\omega)^2 + \gamma^2/4},$$

$$\alpha(\omega) \approx \frac{kNe^2}{4m\epsilon_0\omega_0} \frac{\gamma}{(\Delta\omega)^2 + \gamma^2/4}.$$

(b) Find the values of $\Delta\omega$ for which $n(\omega)$ reaches a local maximum or minimum.

(c) Prove that the absorption coefficient $\alpha(\omega)$ reaches *half* its maximum value at the values of $\Delta\omega$ you found in part (b).