Physics 47 — Optics

Problem Set 8
Due Fri, Nov 10, 2017
(before start of class)

Textbook Problems

1. Hecht 11.17 [4 pts.]
   This is a simple mathematical exercise, but it should also help illustrate the nature of the term in the array theorem that accounts for moving an aperture off-center.

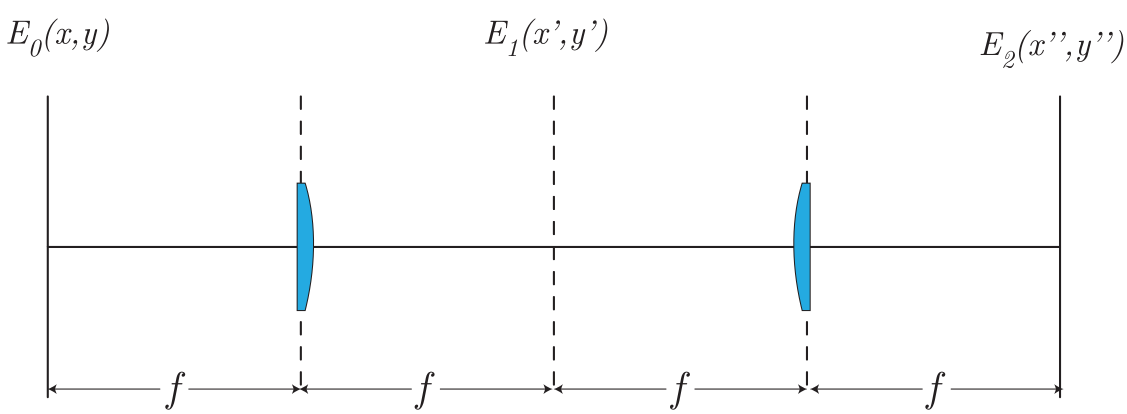
2. Hecht 11.21 [4 pts.]
   This is an "easy" 2D convolution, to test your understanding before tackling 11.49.

3. Hecht 11.29 [8 pts.]
   A proof that is obviously much easier to do if you make use of the convolution theorem. NOTE: $\mathcal{F}\left\{f(x)\sin(k_0 x)\right\} = \left[F(k+k_0) - F(k-k_0)\right]/2i$ (the book's expression should be multiplied times -1). I suspect this error came from using inconsistent conventions for the Fourier transforms. You may notice that this issue is mentioned briefly in Hecht 11.2.1, page 535.

4. Hecht 11.49 [6 pts.]
   This autocorrelation's a step harder than 11.21, but if you're getting stuck you should be able to figure it out from the discussion on pp. 555-556. You only need to make a sketch of the autocorrelation function. Indicate where the edge of the function is and explain the three different regions that occur.

Additional Problems

A1. **Fourier transform of a Fourier Transform** [6 pts.]
Consider the "4-f" imaging system shown below, and suppose that there is nothing between the two lenses of focal length f.



If we make the paraxial approximation and ignore aperture effects, the electric field in the focal plane of the first lens is given by

$$E_1(x', y') = \frac{k}{2\pi f}\tilde{E}_0\left(\frac{k}{f}x', \frac{k}{f}y'\right) = \frac{k}{2\pi f}\iint_{-\infty}^{\infty} E_0(x, y)e^{i\frac{k}{f}(xx'+yy')}\,dx\,dy$$

where $k = 2\pi/\lambda$. Show by computing the Fourier transform (*not* the inverse transform) of the focal-plane field (E_1), that the field in the output plane is of the form

$$E_2(x'', y'') = CE_0(-x, -y)$$

where C is a constant. Strictly speaking, this is true only in the paraxial approximation. We have been generally neglecting this issue of image inversion, but you should have noticed this while working on the Fourier Optics lab. The point is, the transformation of the field going to point 2 is another Fourier transform, not an inverse Fourier transform.